WOBURN CHALLENGE

2018-19 Online Round 4

Friday, March 22nd, 2019

Senior Division Problems

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Problem S1: World of StarCraft

14 Points / Time Limit: 3.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc184s1*

Billy, the king of video games, is (among other things) a top professional *StarCraft II* player. In light of his recent success as the only human to defeat the fearsome AlphaStar AI, he has received exclusive access to play the upcoming installment in the series, *World of StarCraft*. In a natural progression for the series, this is a massively multiplayer online role-playing game.

World of StarCraft features N ($2 \le N \le 100{,}000$) planets, numbered from 1 to N. Each planet is under the control of one of three races (Protoss, Terran, or Zerg), with the character R_i representing planet i's race (either "P" for Protoss, "T" for Terran, or "Z" for Zerg).

There are M ($0 \le M \le 100{,}000$) space routes running amongst the planets, with the i-th space route allowing one to travel in either direction between planets A_i and B_i ($1 \le A_i, B_i \le N$, $A_i \ne B_i$). There may be multiple space routes connecting a single pair of planets.

Despite its secretive and unreleased development status, *World of StarCraft* somehow already has an enormous player-base. Billy himself has K ($1 \le K \le 100{,}000$) friends in the game. His i-th friend is currently on planet X_i, and would like to reach a different planet Y_i to complete an objective ($1 \le X_i, Y_i \le N$; $X_i \ne Y_i$).

When a player is on a planet i, they may travel along a space route to another planet j, assuming that a space route exists which connects planets i and j. However, due to open hostilities amongst the three races, it's only safe to do so if both planets are under the control of the same race (in other words, if $R_i = R_j$).

Billy would love to help all of his friends complete their objectives, but not without putting their in-game lives at risk! Help him determine how many friends i he has such that it's possible to safely travel through a sequence of planets beginning at planet X_i and ending at planet Y_i.

Subtask

In test cases worth 8/14 of the points, $N \le 1000$, $M \le 1000$, and $K \le 1000$.

Input Format

The first line of input consists of three space-separated integers, N, M, and K.
The next line consists of characters, $R_{1..N}$.
M lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..M$.
K lines follow, the i-th of which consists of two space-separated integers, X_i and Y_i, for $i = 1..K$.

Output Format

Output a single integer, the number of friends who can safely complete their objectives.

Sample Input

```
6 7 3
PZZPPP
1 2
6 4
3 4
2 5
5 6
1 4
4 2
6 4
2 3
1 5
```

Sample Output

```
2
```

Sample Explanation

Billy's first friend can safely travel directly from planet 6 to planet 4 to complete their objective.

Billy's second friend may never safely reach planet 3.

Billy's third friend can safely travel through the sequence of planets 1 → 4 → 6 → 5.

Problem S2: Farming Simulator

19 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc184s2*

When Billy needs an extra dose of excitement in his life, he looks no further than to one of his favourite video game series, *Farming Simulator*. It's filled with wonderful games, though *Farming Simulator 16* takes the cake.

Billy's farm in *Farming Simulator 16* includes a certain long stretch of dirt, which can be represented as a number line. Billy's currently standing at position 0 along the stretch, while his farmhouse is at position H ($2 \le H \le 500{,}000{,}000$). He has also prepared N ($1 \le N \le 3000$) holes in the dirt, the i-th of which is at a distinct position P_i ($1 \le P_i \le H - 1$).

Billy can walk along the stretch in either direction at a speed of 1 unit per second, and he may also choose to stand still at any point. He'd like to walk around and get a tree growing in each of the N holes, before ending up at his farmhouse!

Getting a tree growing in the i-th hole is a two-step process. First, at any point in time when Billy is at position P_i, he may instantly plant a seed in the hole. Then, at any point in time at least W_i ($1 \le W_i \le 500{,}000{,}000$) seconds after the seed has been planted, when Billy is once again at position P_i, he may instantly water the seed to make it start growing. Note that the minimum waiting time W_i may differ from hole to hole, due to realistically complex details of dirt composition.

Help Billy determine the minimum amount of time required for him to begin at position 0, get a tree growing in each of the N holes, and then end up at position H.

Subtask

In test cases worth 14/19 of the points, $N \le 200$.

Input Format

The first line of input consists of two space-separated integers, N and H.
N lines follow, the i-th of which consists of two space-separated integers, P_i and W_i, for $i = 1..N$.

Output Format

Output a single integer, the minimum number of seconds required for Billy to complete his task.

Sample Input

```
3 10
7 3
8 1
4 2
```

Sample Output

```
15
```

Sample Explanation

One optimal strategy for Billy is as follows:

- Walk to position 4, and plant a seed in hole 3 (4s)
- Stand still for 2 seconds, and water the seed in hole 3 (2s)
- Walk to position 7, and plant a seed in hole 1 (3s)
- Walk to position 8, and plant a seed in hole 2 (1s)
- Walk to position 7, stand still for 1 second, and water the seed in hole 1 (2s)
- Walk to position 8, and water the seed in hole 2 (1s)
- Walk to position 10 (2s)

Problem S3: Dance Royale

28 Points / Time Limit: 5.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc184s3*

Billy is trying his hand at the latest popular game taking the world by storm: *Dance Royale*.

In *Dance Royale*, there are N ($1 \le N \le 300{,}000$) locations on a map (numbered from 1 to N). Each location i has a destination number D_i ($0 \le D_i \le N$, $D_i \ne i$), which is used during gameplay (as described below).

There are also M ($2 \le M \le 300{,}000$) players, with the i-th player beginning the game at location L_i ($1 \le L_i \le N$). Each player has some sick dance moves.

The game proceeds in sets of three phases as follows:

1. For each unordered pair of players still in the game, if they are currently at the same location and have not yet had a dance-off against one another, then they engage in a dance-off against one another. Nobody is harmed in the process, a good time is simply had.
2. For each player still in the game, let d be their current location's destination number. If $d = 0$, then they're forced to permanently leave the game. Otherwise, they move to location d.
3. If there are fewer than 2 players left in the game, then the game ends. Otherwise, the process repeats itself from phase 1.

Note that the game may last forever, which is fine — Billy is accustomed to extended periods of mental focus.

After the game has either ended or has gone on for an infinite amount of time, how many dance-offs will end up having taken place in total?

Subtasks

In test cases worth 6/28 of the points, $N \le 50$, $M \le 50$, and $D_i > 0$ for each i.
In test cases worth another 6/28 of the points, $N \le 2000$, and $D_i > 0$ for each i.
In test cases worth another 10/28 of the points, $D_i > 0$ for each i.

Input Format

The first line of input consists of two space-separated integers, N and M.
N lines follow, the i-th of which consists of a single integer, D_i, for $i = 1..N$.
M lines follow, the i-th of which consists of a single integer, L_i, for $i = 1..M$.

Output Format

Output a single integer, the number of dance-offs which will take place.

Sample Input 1

```
4 4
4
3
1
3
4
2
3
4
```

Sample Output 1

```
3
```

Sample Input 2

```
5 6
4
0
4
1
1
4
2
5
3
2
2
```

Sample Output 2

```
4
```

Sample Explanation

In the first case:

- Right off the bat, a dance-off will occur between players 1 and 4, as they both occupy location 4.
- Then, in the second cycle of the phases, players 1, 2, and 4 will all find themselves at location 3, resulting in player 2 having dance-offs with both players 1 and 4. Note that players 1 and 4 will not repeat their dance-off against one another.
- The game will end up continuing forever with all 4 players in action, but no more dance-offs will ever take place.

In the second case:

- Right off the bat, dance-offs will occur between player pairs (2, 5), (2, 6), and (5, 6), due to players 2, 5, and 6 all occupying location 2. These 3 players will then leave the game in phase 2.
- Then, in the second cycle of the phases, players 1 and 3 will both find themselves at location 1 and will therefore have a dance-off.
- The game will end up continuing forever with 3 players remaining, but no more dance-offs will ever take place.

Problem S4: Super Luigi Odyssey

39 Points / Time Limit: 7.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc184s4*

Billy has been having a great time playing a demo of Nintendo's next highly-anticpated 3D platforming game, *Super Luigi Odyssey*.

One challenge in the game sees Luigi trapped in a long hallway, which can be represented as a number line with positions increasing towards the rightwards direction. There are N ($1 \le N \le 250{,}000$) platforms in it, with the i-th one at position P_i ($0 \le P_i \le 10^9$) and at a height of H_i ($1 \le H_i \le 10^9$) metres. No two platforms are at the same position. Luigi begins on platform 1 (note that this is not necessarily the leftmost platform).

Much to Luigi's concern, the hallway is filled with some deadly lava. Initially, the lava reaches up to a height of 0.5 metres. At any point, a platform is considered to be submerged in lava if the lava's height exceeds the platform's height.

A sequence of M ($1 \le M \le 250{,}000$) events will then occur, each having one of three possible types. The type of the i-th event is described by the integer E_i ($1 \le E_i \le 3$).

- If $E_i = 1$, then the lava's height will increase by X_i ($-10^9 \le X_i \le 10^9$) metres. It's guaranteed that this will not cause the lava's height to become negative. If this causes Luigi's current platform to become submerged, then he will immediately perish.
- If $E_i = 2$, then X_i ($1 \le X_i \le N$) lasers in a row will be fired at Luigi. Each laser will force him to jump to the next non-submerged platform to the left of his current one. If there's no such platform, then he'll instead be forced to jump into the lava and perish.
- If $E_i = 3$, then similarly X_i ($1 \le X_i \le N$) lasers in a row will be fired at Luigi, with each one forcing him to jump to the next non-submerged platform (if any) to the right rather than the left.

Luigi is not allowed to move between platforms aside from being forced to by type-2 or type-3 events.

Even if Billy manages to keep Luigi alive through all M events, he may not be out of the woods yet — his success in later challenges will depend on how much of Luigi's energy has been spent. Whenever Luigi jumps from platform i to platform j, he expends $|P_i - P_j|^K$ ($1 \le K \le 2$) units of energy. Note that the amount of energy required doesn't depend on the platforms' relative heights.

Help Billy determine how much energy Luigi will expend throughout all M events (if he will even survive that long). As this may amount to quite a few units of energy, you only need to determine the total modulo 1,000,000,007.

Subtasks

In test cases worth 6/39 of the points, $N \le 2000$, $M \le 2000$, and $K = 1$.
In test cases worth another 16/39 of the points, $K = 1$.

Input Format

The first line of input consists of three space-separated integers, N, M, and K.
N lines follow, the i-th of which consists of two space-separated integers, P_i and H_i, for i = 1..N.
M lines follow, the i-th of which consists of two space-separated integers, E_i and X_i, for i = 1..M.

Output Format

Output a single integer, the total number of units of energy which Luigi will expend (modulo 1,000,000,007), or −1 if he will be forced to touch the lava and perish at any point.

Sample Input 1

```
5 7 1
4 4
5 5
13 6
0 8
10 8
3 1
1 4
2 1
1 -1
3 4
1 2
2 2
```

Sample Output 1

```
32
```

Sample Input 2

```
2 2 2
0 2
1 1
1 1
3 1
```

Sample Output 2

```
-1
```

Sample Explanation

In the first case, Luigi will be forced to jump along the follow sequence of positions:

- Event 1: 4 → 5
- Event 3: 5 → 0
- Event 5: 0 → 4 → 5 → 10 → 13
- Event 7: 13 → 10 → 0

In total, these jumps require 32 units of energy (which is equal to 32 modulo 1,000,000,007). If K were equal to 2 rather than 1, then 186 units of energy would be required instead.

In the second case, after the lava's height is raised to 1.5 metres, Luigi will have no non-submerged platform to jump to on his right, and so will be forced to jump into the lava and perish.