WOBURN CHALLENGE

2018-19 Online Round 3

Friday, February 1st, 2019

Senior Division Problems

Automated grading is available for these problems at:
wcipeg.com

For more problems from past contests, visit:
woburnchallenge.com

Problem S1: The Perfect Team

14 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc183s1*

Jessie, James, and Meowth, members of the honourable Team Rocket, have finally hit the jackpot! They've managed to steal a group of N ($1 \le N \le 300{,}000$) Pokémon. There are K ($1 \le K \le N$) different Pokémon types, numbered from 1 to K. The i-th Pokémon has type T_i ($1 \le T_i \le K$), and level L_i ($1 \le L_i \le 10^9$). There's at least one Pokémon of each type 1..K.

What remains is for Team Rocket to make the best use of their haul. They can't necessarily afford to carry around that many Pokémon with them, so they'd like to choose exactly M ($K \le M \le N$) of the N Pokémon to form an unstoppable battling team. Putting all of the highest-level Pokémon to use would be nice, but Team Rocket's top priority is putting together a team which has no glaring weaknesses. To make sure they're covered against anything they might face, they insist that their M-Pokémon team must include at least one Pokémon of each type 1..K.

Subject to those conditions, help Team Rocket determine the maximum sum of Pokémon levels which such a team could possibly have! Please note that the answer may not fit within a 32-bit signed integer.

Subtasks

In test cases worth 7/14 of the points, $M = K$.

Input Format

The first line of input consists of three space-separated integers, N, M, and K.
N lines follow, the i-th of which consists of two space-separated integers, T_i and L_i, for i = 1..N.

Output Format

Output a single integer, the maximum sum of Pokémon levels which a valid team of M Pokémon can have.

Sample Input 1

```
5 3 3
1 8
2 5
1 13
3 5
2 4
```

Sample Output 1

```
23
```

Sample Input 2

```
7 5 2
1 11
2 10
1 16
2 11
1 19
1 7
2 15
```

Sample Output 2

```
72
```

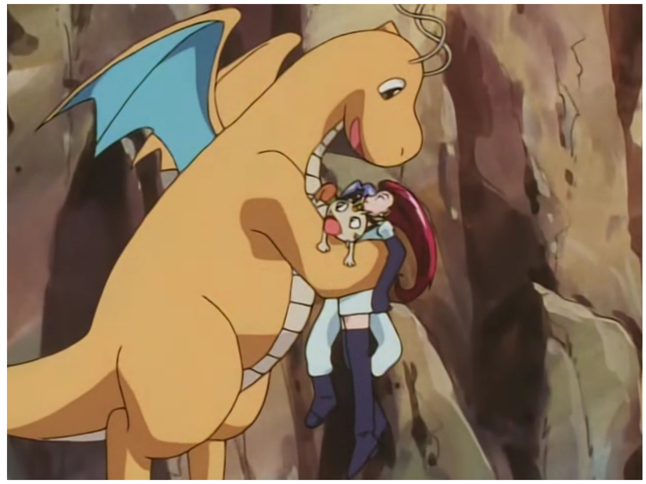
Sample Explanation

In the first case, the 2nd, 3rd, and 4th Pokémon should be chosen. All Pokémon types 1..3 are represented, and the sum of these Pokémon's levels is 5 + 13 + 5 = 23, which is the largest achievable sum.

In the second case, the 1st, 3rd, 4th, 5th, and 7th Pokémon should be chosen.

Problem S2: Gym Tour

20 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc183s2*



It's every Pokémon trainer's dream to visit all of the Pokémon gyms in their region, defeating each of their gym leaders and claiming gym badges proving their worth! The members of Team Rocket also dream of visiting all of the gyms, though that's mostly because they're sure to be filled with powerful Pokémon worth stealing…

A certain region has N ($2 \le N \le 100{,}000$) towns, numbered from 1 to N. There are $N - 1$ routes running amongst the towns, each of which allows travelers to walk in either direction between a pair of towns, such that each town may be reached from any other town by following a sequence of one or more routes. The i-th route runs between towns A_i and B_i ($1 \le A_i, B_i \le N$, $A_i \ne B_i$).

There are K ($1 \le K < N$) Pokémon gyms in the region, the i-th of which is located in town G_i ($2 \le G_i \le N$). No two gyms are in the same town, and none of the gyms are in town 1.

Team Rocket would like to pay a friendly visit to each gym's town at least once. They currently find themselves in town 1, and it takes them one whole day to walk along a route from their current town to another town. It takes them no time to conduct any business in the gyms, but all of this walking looks to be very time-consuming by itself!

Fortunately, another potential method of travel is also available to them. A winged Pokémon named Dragonite can be found in town F ($1 \le F \le N$), and Team Rocket may be able to enlist its transportation services. If they're ever in town F, they may choose to catch Dragonite. Anytime after they've caught Dragonite, they may choose to Fly back to any town they've previously visited. Catching Dragonite and Flying to a previous town each take no time at all. It's guaranteed that Dragonite is not at a town which has a Pokémon gym.

What's the minimum number of days required for Team Rocket to visit all K gyms after beginning in town 1?

Subtasks

In test cases worth 9/20 of the points, $F = 1$.

Input Format

The first line of input consists of three space-separated integers, N, K, and F.
The next line consists of integers, $G_{1..K}$.
$N - 1$ lines follow, the i-th of which consists of two integers, A_i and B_i, for $i = 1..(N - 1)$.

Output Format

Output a single integer, the minimum number of days required for Team Rocket to visit all K gyms.

Sample Input 1

```
4 3 1
3 4 2
1 2
3 2
4 2
```

Sample Output 1

```
3
```

Sample Input 2

```
5 2 4
2 5
1 2
2 3
3 4
1 5
```

Sample Output 2

```
3
```

Sample Input 3

```
7 3 2
7 6 3
5 6
4 7
2 4
1 4
6 1
4 3
```

Sample Output 3

```
5
```

Sample Explanation

In the first case, Team Rocket can immediately catch Dragonite in town 1. They can then walk to town 2 followed by town 3, visiting both of their gyms. They can then Fly back to town 2, and finally walk from there to town 4 to visit its gym. In total, they will have walked from town to town 3 times, resulting in the whole tour taking 3 days.

In the second case, Team Rocket can begin by walking to town 2 and visiting its gym. They can then walk back to town 1 and then to town 5, visiting its gym as well.

Problem S3: Counterpicking

26 Points / Time Limit: 10.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc183s3*

Jessie's been training hard, and is ready to go out there and claim some new Pokémon for Team Rocket fair and square! Or at least, by beating some Pokémon trainers in battles fair and square, and then proceeding to steal their Pokémon.

Jessie has N ($1 \le N \le 300{,}000$) Pokémon at her disposal, with the i-th one having two traits of interest — a Strength of A_i and a Speed of B_i ($1 \le A_i, B_i \le 10^9$).

She'll be battling against M ($1 \le M \le 300{,}000$) trainers, one after another, each with a single Pokémon. In each battle, Jessie will choose one of her N Pokémon to use in it. There are no restrictions regarding her choices — for example, she may choose to use any of her Pokémon either in multiple battles, or not at all.

Each trainer's Pokémon is vulnerable against the Strength and Speed traits to various degrees, based on the values X_i and Y_i ($1 \le X_i, Y_i \le 10^9$). Battle Effectiveness is a measure of how effective Jessie's chosen Pokémon is against a trainer's Pokémon. If Jessie chooses to use her j-th Pokémon in the i-th battle, the resulting Battle Effectiveness will be $X_i \times A_j + Y_i \times B_j$.

For each of the M battles, help Jessie determine the maximum possible Battle Effectiveness that she can achieve by choosing an optimal Pokémon to use. Please note that the answer may not fit within a 32-bit signed integer.

Subtasks

In test cases worth 6/26 of the points, $N \le 2000$ and $M \le 2000$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..N$.
The next line consists of a single integer, M.
M lines follow, the i-th of which consists of two space-separated integers, X_i and Y_i, for $i = 1..M$.

Output Format

M lines, the i-th of which is the maximum Battle Effectiveness which Jessie can achieve in the i-th battle

Sample Input

```
2
1 10
8 2
2
10 1
1 10
```

Sample Output

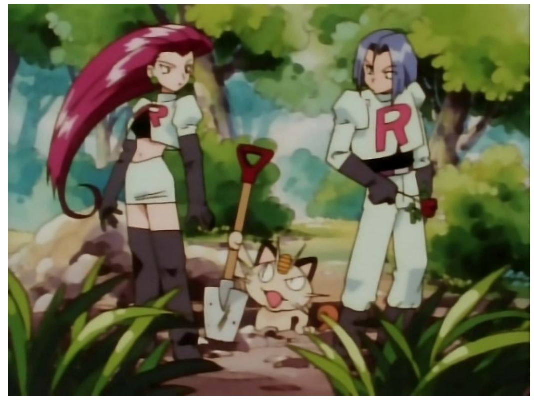
```
82
101
```

Sample Explanation

In the first battle, Jessie should use her second Pokémon for a Battle Effectiveness of 10×8 + 1×2 = 82. In the second battle, she should instead use her first Pokémon for a Battle Effectiveness of 1×1 + 10×10 = 101.

Problem S4: Holey Travels

40 Points / Time Limit: 5.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc183s4*



When it comes down to it, there's nothing that the members of Team Rocket enjoy more than the simple, familiar things in life. In other words, digging holes to trap Pokémon trainers and steal their Pokémon.

Today, N ($1 \le N \le 1000$) Pokémon trainers will be traveling over an open field which can be represented as an infinite 2D plane. The i-th Pokémon trainer will be walking along the infinite, straight line which passes through two distinct points ($X1_i$, $Y1_i$) and ($X2_i$, $Y2_i$) ($-10{,}000 \le X1_i, Y1_i, X2_i, Y2_i \le 10{,}000$, $(X1_i, Y1_i) \ne (X2_i, Y2_i)$). Multiple trainers may walk along exactly the same path. Note that each path extends infinitely in both directions (it's neither a line segment nor a ray). Team Rocket don't care in which direction along this path the trainer will be walking. What they do care about is the fact that the i-th trainer will have P_i ($1 \le P_i \le 1{,}000{,}000$) Pokémon with them!

Jessie, James, and Meowth have time to dig a single hole somewhere on this infinite field before the trainers begin walking along their paths. This hole will be a circle with real-valued radius R ($1 \le R \le 100{,}000$), centered at any point (with real-valued coordinates) of their choice.

Once the hole has been dug, each trainer whose path turns out to intersect with or touch the hole's circle (inclusively) will fall into it, with all of their Pokémon becoming trapped in the hands of Team Rocket. What's the maximum number of Pokémon which Team Rocket can trap by choosing an optimal excavation location?

Subtasks

In test cases worth 8/40 of the points, $N \le 2$.
In test cases worth another 20/40 of the points, $N \le 100$.

Input Format

The first line of input consists of a single integer, N, and 1 real number, R.
N lines follow, the i-th of which consists of five space-separated integers, $X1_i$, $Y1_i$, $X2_i$, $Y2_i$, and P_i, for i = 1..N.

Output Format

Output a single integer, the maximum number of Pokémon which Team Rocket can trap. It's guaranteed that increasing or decreasing R by up to 10^{-5} would not change the answer.

Sample Input 1

```
4 3.0
3 0 5 4 3
-2 5 7 0 8
-5 -5 7 1 9
1 6 -7 1 12
```

Sample Output 1

```
23
```

Sample Input 2

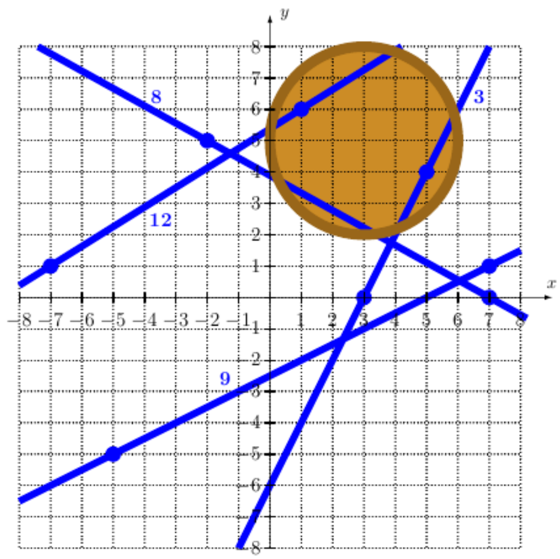
```
5 2.1
2 1 6 1 2
3 -2 -5 -2 3
7 5 1 5 2
-5 -3 -4 -3 1
-6 -7 4 -7 4
```

Sample Output 2

```
6
```

Sample Explanation

The following diagram illustrates the first case, with the Pokémon trainers' paths indicated in blue, and one possible optimal hole location (trapping 3 + 8 + 12 = 23 Pokémon) indicated in brown:



The second case is similarly illustrated below: