WOBURN CHALLENGE

2018-19 On-Site Finals

Saturday, May 18th, 2019

Senior Division Problems

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wcipeg.com

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Problem S1: Behind the Scenes

9 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc18fs1*

The cows and monkeys of Scarberia, their long-standing conflicts well behind them at last, have banded together to produce a historical drama documenting their past battles. With the Head Monkey as writer and producer, and the director's chair occupied by the cows' leader, Bo Vine, this will be a collaborative masterpiece to remember!

Today, a full day of filming is set to take place at the monkeys' and cows' joint studios. The studios feature three sound stages, numbered from 1 to 3, and there are initially E_i ($0 \le E_i \le 1000$) pieces of filmmaking equipment on stage i.

A sequence of N ($1 \le N \le 1000$) shots will be filmed, one after another, with the i-th one taking place on stage S_i ($1 \le S_i \le 3$). Whenever a shot is filmed on a certain stage, there must be no equipment present on that stage at the time. Right before each shot, Bo Vine may choose 0 or more pieces of equipment to be moved. Each such piece will be moved from its current stage to a different stage of Bo's choice, at the cost of tipping a stagehand $1, and will remain there until potentially moved again later.

What's the minimum total cost of stagehand tips which Bo Vine will need to dispense such that each of the N shots will have no equipment present on its stage at filming time?

Input Format

The first line of input consists of three space-separated integers, E_1, E_2, and E_3.
The next line consists of a single integer, N.
N lines follow, the i-th of which consists of a single integer, S_i, for $i = 1..N$.

Output Format

Output a single integer, the minimum total cost of stagehand tips required, in dollars.

Sample Input

```
6 3 4
5
3
2
2
1
2
```

Sample Output

```
17
```

Sample Explanation

One possible optimal strategy for Bo Vine is as follows:

- Before the 1st shot, move all 4 pieces of equipment from stage 3 to stage 2.
- Before the 2nd shot, move all 6 pieces of equipment from stage 1 to stage 3, and all 7 pieces of equipment from stage 2 to stage 3.

This requires $4 + $6 + $7 = $17 in stagehand tips.

Problem S2: Top Billing

14 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc18fs2*

Two particularly famous actors are starring in the cows' and monkeys' production: Tom Cows and Monkey Freeman. With both of them playing equally important roles, one question is at the forefront of everybody's mind: which of them will receive top billing in the credits and get to be the main face of the movie?

Tom isn't one to stand for playing second fiddle, so he's hatching a plan to discredit Monkey in the eyes of the director, Bo Vine. He's going to sneak into the main studio building in the middle of the night and rearrange its layout, in the hopes of causing Monkey to be late for the following morning's filming session. That'll show him!

The building can be represented as a grid with R rows and C columns ($2 \le C \le R \le 100$). Note that the grid has at least as many rows as it has columns. Tom will choose one starting cell (leading to the parking lot) and one different ending cell (leading to the movie set). These cells may be anywhere in the grid (not necessarily on its edges). For each remaining cell in the grid, he'll choose to either leave it vacant or obstruct it with a wall. He'll make sure that it's possible to reach the ending cell from the starting one through a sequence of horizontally/vertically adjacent vacant cells.

In the morning, Tom and Monkey will begin in the starting cell and will each attempt to make their way to the ending cell, by repeatedly moving from cell to adjacent cell (up, right, down, or left) at a rate of 1 minute per cell. Tom, having arranged the building, will follow a route allowing him to reach the ending cell as quickly as possible. Monkey, on the other hand, will follow the following procedure:

1. Make an ordered list of possible next cells to move to as follows: for each of the directions up/right/down/left (in that order), if the cell adjacent to his current one in that direction is within the grid and doesn't contain a wall, include it in the list. Note that this list will always contain between 1 and 4 cells, inclusive.
2. Move to the cell in the list which is closest to the ending cell by Manhattan distance* (ignoring any walls). If multiple cells in the list have the same minimum Manhattan distance to the ending cell, choose the one of them which appears earliest in the list.
3. If he's arrived at the ending cell, stop. Otherwise, return to Step 1.

** The Manhattan distance between a cell in (row r_1, column c_1) and another cell in (row r_2, column c_2) is defined as $|r_1 - r_2| + |c_1 - c_2|$.*

For example, consider the following two building layouts (with "`S`" denoting the starting cell, "`E`" denoting the ending cell, and walls marked with "`#`"s):

```
...S
.##.
.##.
E...
```

Monkey would reach the ending cell after 6 minutes (following the sequence of moves ↓↓↓←←←).

```
...S
....
....
E.#.
```

Monkey would walk around forever (following the sequence of moves ↓↓↓↑↓↑↓↑↓…).

Help Tom come up with any building layout which would cause Monkey to successfully arrive the ending cell from the starting one eventually (rather than walking around forever as in the second example above), but at least $2C - 4$ minutes after Tom does. It's guaranteed that at least one such building layout exists for any valid pair of dimensions R and C (with $C \le R$).

Subtasks

In test cases worth 6/14 of the points, $R \le 4$ and $C \le 4$.

Input Format

The first and only line of input consists of two space-separated integers, R and C.

Output Format

Output R lines of C characters each, a grid representing the chosen building layout. Each character must be one of the following:

- "`S`": starting cell (which must appear exactly once)
- "`E`": ending cell (which must appear exactly once)
- "`.`": vacant cell
- "`#`": wall

Sample Input 1

```
4 2
```

Sample Output 1

[Multiple possible accepted grids]

Sample Input 2

```
10 9
```

Sample Output 2

[Multiple possible accepted grids]

Sample Explanation

In the first case, below is an example of a grid which would be accepted:

```
ES
..
.#
#.
```

For this grid, both Tom and Monkey would reach the ending cell in 1 minute, by moving to the left. This is a difference of 0, which is at least as large as the minimum required difference of $2C - 4 = 0$. Therefore, this output would be judged as *correct*.

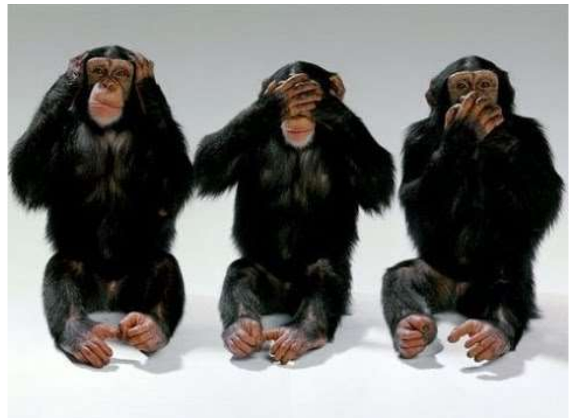
In the second case, below is an example of a grid which satisfies the basic output format requirements, but would *not* be accepted:

```
.........
.........
.........
.........
.........
.........
.........
....#....
.S#...#E.
.........
```

For this grid, Monkey would reach the ending cell in 10 minutes (following the sequence of moves ↑→→↓→→↑→→↓), while Tom would only require 8 minutes (following the sequence of moves ↓→→→→→→↑). This is a difference of 2, which is smaller than the minimum required difference of $2C - 4 = 14$. Therefore, this output would be judged as *incorrect*. Various other grids exist which would be judged as *correct*, however they are not disclosed here.

Problem S3: Screen Time

17 Points / Time Limit: 5.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc18fs3*



A trio of rival actors, numbered 1..3 from oldest to youngest, are set to make appearances in the cows' and monkeys' film. However, their contracts state that the oldest of them (actor 1) must receive more screen time than either one of the others! The Head Monkey may need to adjust her script to ensure that that's the case.

There are N ($1 \le N \le 200{,}000$) possible equal-length scenes to include in the script, each of which involves exactly two of the three actors in question. The i-th scene features the two actors A_i and B_i ($1 \le A_i, B_i \le 3$, $A_i \ne B_i$). The Head Monkey may select any non-empty subset of these N scenes to include (note that there are $2^N - 1$ such possible subsets).

Letting S_i be the number of scenes featuring actor i, how many different subsets of scenes could the Head Monkey choose to include such that both $S_1 > S_2$ and $S_1 > S_3$ hold? As there may be many valid subsets, you only need to compute the number of them modulo 1,000,000,007.

Subtasks

In test cases worth 4/17 of the points, $N \le 20$.
In test cases worth another 3/17 of the points, $N \le 200$.
In test cases worth another 3/17 of the points, $N \le 3000$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..N$.

Output Format

Output a single integer, the number of valid subsets of scenes (modulo 1,000,000,007).

Sample Input 1

```
5
3 1
1 2
3 2
2 3
3 1
```

Sample Output 1

```
3
```

Sample Input 2

```
15
3 2
1 2
3 1
2 1
2 1
2 3
1 3
3 2
1 2
1 3
2 3
3 1
1 3
2 3
2 1
```

Sample Output 2

```
7266
```

Sample Explanation

In the first case, if the subset of scenes (1, 2) is chosen, then $S_1 = 2$ while $S_2 = S_3 = 1$, making this a valid subset. Subsets (1, 2, 5) and (2, 5) are also valid. Therefore, the answer is 3 modulo 1,000,000,007 = 3.

Problem S4: Posters

26 Points / Time Limit: 4.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc18fs4*

With principal photography on the cows' and monkeys' project having concluded, it won't be long now before post-production wraps up and the film can be shown to the public! In preparation for the initial run of screenings, the Head Monkey and Bo Vine are taking it upon themselves to personally put up movie posters at movie theatres all across the land of Ontario.

There are N ($3 \le N \le 300$) cities in Ontario, numbered from 1 to N. City i either has a movie theatre (indicated by C_i = "`1`") or doesn't (C_i = "`0`"), and at least one city in Ontario has a theatre.

There are also $N - 1$ highways, the i-th of which allows one to travel in either direction between two different cities X_i and Y_i ($1 \le X_i, Y_i \le N$, $X_i \ne Y_i$). It's possible to travel from any city to any other city by following a sequence of highways.

The Head Monkey will begin in city A, while Bo Vine will begin in a different city B ($1 \le A, B \le N$, $A \ne B$). Neither of their starting cities have movie theatres ($C_A = C_B$ = "`0`"). Each hour, the Head Monkey and/or Bo Vine may each travel along a highway from their current city to an adjacent one. They are allowed to both be present in the same city as one another. Whenever either of them visits a city with a movie theatre, they may put up a poster at it, which requires no additional time.

Between the two of them, the Head Monkey and Bo Vine would like to put up a poster at every movie theatre in Ontario. In other words, for each city i such that C_i = "`1`", at least one of the two must visit it at some point. Furthermore, the Head Monkey and Bo Vine would each like to end up back at their original cities of A and B, respectively. Time is of the essence when it comes to getting the word out about their film, so help them determine the minimum amount of time required to complete this task!

Subtasks

In test cases worth 8/26 of the points, $N \le 50$, and each city has at most two highways directly adjacent to it.
In test cases worth another 11/26 of the points, $N \le 50$.

Input Format

The first line of input consists of three space-separated integers, N, A, and B.
The next line consists of N characters, $C_{1..N}$.
$N - 1$ lines follow, each of which consists of two space-separated integers, X_i and Y_i, for $i = 1..(N - 1)$.

Output Format

Output a single integer, the minimum number of hours required for the Head Monkey and Bo Vine to visit all of the theatres and each return to their original city.

Sample Input 1

```
5 2 4
00101
1 2
2 3
3 4
4 5
```

Sample Output 1

```
2
```

Sample Input 2

```
8 1 6
00110011
1 2
2 3
2 4
4 5
1 6
6 7
7 8
```

Sample Output 2

```
6
```

Sample Input 3

```
5 1 2
00101
5 4
3 1
1 2
4 1
```

Sample Output 3

```
4
```

Sample Explanation

In the first case, one possible optimal pair of routes is as follows:

- Head Monkey: 2 → 3 → 2
- Bo Vine: 4 → 5 → 4

In the second case, one possible optimal pair of routes is as follows:

- Head Monkey: 1 → 2 → 3 → 2 → 4 → 2 → 1
- Bo Vine: 6 → 7 → 8 → 7 → 6

In the third case, one possible optimal pair of routes is as follows:

- Head Monkey: 1 → 4 → 5 → 4 → 1
- Bo Vine: 2 → 1 → 3 → 1 → 2

Problem S5: Opening Weekend

34 Points / Time Limit: 8.00s / Memory Limit: 256M *Submit online: http://wcipeg.com/problem/wc18fs4*

At last, the time has come for months of hard work to finally pay off — the cows' and monkeys' production is hitting the big screen this weekend!

It will be playing at movie theatres in N ($2 \le N \le 400{,}000$) different cities, numbered from 1 to N in increasing order of their theatres' quality. There are $N - 1$ roads running amongst these cities, the i-th of which allows vehicles to drive in either direction between cities A_i and B_i ($1 \le A_i, B_i \le N$, $A_i \ne B_i$).

However, each road also passes through a tunnel which imposes a limit on the heights of vehicles which may drive along it — in particular, a vehicle may only travel along the i-th road if its height is at most L_i cm ($1 \le L_i \le 10^9$). It's possible for a 1cm-high vehicle (if such a thing exists) to travel from any city to any other city by following a sequence of roads.

There are K ($1 \le K \le 400{,}000$) moviegoers who have been waiting anxiously to see the film, with the i-th one living in city C_i ($1 \le C_i \le N$) and driving a vehicle with a height of H_i cm ($1 \le H_i \le 10^9$). However, they won't necessarily settle for watching the film in their own cities — they're prepared to drive wherever it takes to get the best possible movie-watching experience! However, they're also not great at planning ahead, so each moviegoer will follow this simple procedure:

1. They'll consider both their current city and all cities they can directly reach from that city (by driving along a single road whose tunnel their vehicle can fit through), and find the one with the highest-quality movie theatre (the largest city number).
2. If that largest-numbered city is their current city, they'll stop to watch the film there.
3. Otherwise, they'll drive to that largest-numbered city, and repeat the procedure from Step 1.

The Head Monkey and Bo Vine want to make sure that each theatre has enough seats available for its screening — nobody should be left out from witnessing their masterpiece! In order to help estimate audience sizes, determine which city each of the K moviegoers will end up stopping at to watch the film.

Subtasks

In test cases worth 5/34 of the points, $N \le 2000$ and $K \le 2000$.
In test cases worth another 12/34 of the points, each city has at most two roads directly adjacent to it.

Input Format

The first line of input consists of two space-separated integers, N and K.
$N - 1$ lines follow, the i-th of which consists of three space-separated integers, A_i, B_i, and L_i, for $i = 1..(N - 1)$.
K lines follow, the i-th of which consists of two space-separated integers, C_i and H_i, for $i = 1..K$.

Output Format

Output K lines, the i-th of which should consist of a single integer, the city at which moviegoer i will stop to watch the film, for $i = 1..K$

Sample Input

```
3 3
1 2 10
3 1 5
1 4
2 1
1 10
```

Sample Output

```
3
2
2
```

Sample Explanation

The first moviegoer will drive from city 1 to city 3 and then remain there.
The second moviegoer will remain at city 2.
The third moviegoer will drive from city 1 to city 2 and then remain there.