WOBURN CHALLENGE

2015-16 On-Site Finals

Friday, May 6th, 2016

Senior Division Problems

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Problem S1: FuzzBiz

9 Points / Time Limit: 3.00s / Memory Limit: 32M

To win the upcoming battle against the cows, the Head-Monkey will need a great deal of money to purchase supplies and ammunition. Alas, the monkeys have been in hiding for so many years that they were never able to find jobs and make money. To make up for this financial deficit, the Head-Monkey has decided to run a business on the side. Unfortunately, the monkeys do not possess many belongings that are in demand by the market... except for one – monkey fur is one of the most luxurious and prized materials in the fashion industry! The smooth, silky locks of any monkey's fur are in high demand by the world's finest coat designers and couturiers. In a stroke of genius, the Head-Monkey ordered all of her subordinates to shave off the fuzz on their bodies and donate it to the business. Before long, sales seemed to be booming – and yet, there were still some problems.

Monkeys aren't particularly renowned for their arithmetic. As sales grew, accounting errors were slipping through the cracks, resulting in unnecessary losses to their income. To practice their division skills, the Head-Monkey proposed that her accountants play the classic game of *FizzBuzz*. In FizzBuzz, players sit around in a circle and take turns counting incrementally, replacing any number divisible by three with the word "fizz", and any number divisible by five with the word "buzz". Numbers divisible by both three and five are replaced with "fizzbuzz". The game ends once a player misspeaks or once they hit N ($1 \le N \le 10^9$). For example, a perfect game of FizzBuzz where $N = 16$ would sound like this:

1, 2, fizz, 4, buzz, fizz, 7, 8, fizz, buzz, 11, fizz, 13, 14, fizzbuzz, 16

Monkeys aren't particularly renowned for their articulateness. When the game gets long, no monkey is a fan of saying "…, five thousand seven hundred and ninety two, fizz, five thousand seven hundred and ninety four, buzz, …". Thus, the Head-Monkey invented her own variation of the game – FizzBuzzOok – which is streamlined for primates. The rules of FizzBuzzOok are almost identical to FizzBuzz, except that whenever a number is supposed to be said in FizzBuzz, the player should instead say "ook". As such, a perfect game of FizzBuzzOok where $N = 16$ would sound like this:

ook, ook, fizz, ook, buzz, fizz, ook, ook, fizz, buzz, ook, fizz, ook, ook, fizzbuzz, ook

Monkeys aren't particularly renowned for their memory. Sometimes, games of FizzBuzzOok would have to be paused as the monkeys went on banana breaks. When they returned, they would quickly lose track of where they had stopped! However, they would be able to remember what their desired goal of N was, along with what may have been said in the past M ($1 \le M \le 10^5$; $M \le N$) turns. For example, the monkeys may remember that they were going for a game of $N = 16$ turns and vaguely recall the last $M = 3$ turns had been "ook, fizz, ook". In this case, there are two possible positions at which they could have paused – after either turn 4 or 13.

Given a sequence of words representing the past M turns, your task is to determine the number of possible positions at which the accountants could have paused, within a perfect game of N turns. Their recollection of what was said in the past M turns could very well be faulty, in which case the given sequence may not be a contiguous subsequence of the N turns of the game. In that case, you should report 0 as the number of possible positions.

In test cases worth 3/9 of the points, $N \le 1000$ and $M \le 1000$.

Input Format

The first line of input consists of two space-separated integers N and M.

M lines follow, with the i-th of these lines consisting of a single word that is either "ook", "fizz", "buzz", or "fizzbuzz" (for i = 1..M).

Output Format

Output a single integer – the number of possible positions that the game could have been paused at, or 0 if the given sequence of turns will never occur in a game of FizzBuzzOok up to N.

Sample Input 1

```
16 3
ook
fizz
ook
```

Sample Output 1

```
2
```

Sample Input 2

```
16 3
ook
buzz
fizzbuzz
```

Sample Output 2

```
0
```

Problem S2: Hydration

14 Points / Time Limit: 4.00s / Memory Limit: 64M

The monkeys have made a booming business out of selling their fur, and their newfound cash flow has been directed towards purchasing all kinds of dangerous weapons to be used against the cows. While the cows have sensed the general fact that the monkeys are about to strike, they are puzzled by how their enemies could be acquiring the necessary funds. Watching shipments of nondescript boxes move in and out of the monkeys' lair has put the entirety of Old MacDonald's farm on edge. To address the frustrating turn of events, Bo Vine has decided to send an elite group of N ($1 \le N \le 10^6$) cow spies on an important mission. This group of highly trained agents are to infiltrate the Sacred Tree, gathering intelligence about the monkeys' source of income and other vile plans! There's just one problem... he's noticed that the cows are all rather thirsty. Needing to ask for water during this assignment could very well blow their cover, so Bo Vine needs to ensure that they all get a drink before heading out! Fortunately, there are M ($1 \le M \le 10^6$) water troughs available in the barn.

The i-th cow is C_i ($1 \le C_i \le 10^9$) centimetres tall, while the i-th trough is T_i ($1 \le T_i \le 10^9$) centimetres high. Each cow is willing to drink from a trough if it's no taller than them, and no more than K ($0 \le K \le 10^9$) centimetres shorter than them – in other words, cow i can drink from trough j if $C_i - K \le T_j \le C_i$. Each cow will need to drink from a trough of their choice for exactly 1 minute. The cows certainly like their privacy, so during each minute, each trough can be used by at most one cow. With little time left until the start of their mission, Bo Vine needs your help to determine the minimum amount of time required for all of them to get a drink, if it's even possible!

In test cases worth 7/14 of the points, $N \le 100$ and $M \le 100$.

Input Format

The first line of input consists of three space-separated integers N, M, and K.
N lines follow, with the i-th of these lines consisting of a single integer C_i (for i = 1..N).
M lines follow, with the i-th of these lines consisting of a single integer T_i (for i = 1..M).

Output Format

Output a single integer – the minimum number of minutes required for all of the cows to drink from a trough, or −1 if it's impossible.

Sample Input 1

```
4 4 5
13
24
10
13
20
8
14
22
```

Sample Output 1

```
3
```

Sample Explanation 1

The following is one optimal sequence of events:

- 1st minute: The 4th cow drinks from trough 2 and the 2nd cow drinks from trough 4.
- 2nd minute: The 3rd cow drinks from trough 2.
- 3rd minute: The 1st cow drinks from trough 2.

Sample Input 2

```
2 1 0
1
1000000000
99999999
```

Sample Output 2

```
-1
```

Problem S3: Driving Range

19 Points / Time Limit: 3.00s / Memory Limit: 32M

With intel from the Sacred Tree stolen by hydrated bovine spies, times are getting ever more desperate for the monkeys. The Head-Monkey has decided it's time to send out her own elite special agent – Agent Tiny. Tiny has been ordered to journey to Old MacDonald's farm and perform a precision strike against the cows' military headquarters. For this extremely dangerous mission, Tiny is being outfitted with a sophisticated piece of monkey technology – a fully functional car made of branches and leaves, and powered by banana juice. This car will offer him more than enough speed and maneuverability to infiltrate the barn. Also, it can shoot missiles from its front headlights.

In an effort to ensure that Tiny actually accomplishes something useful with his car in the field, the Head-Monkey has instructed him to take it for a spin on the monkeys' specialized driving course. The course will take place in a very large room, which can be represented as a 2D grid of cells, with some targets on the walls. The rows are numbered in increasing order from North to South (starting from 1), and similarly the columns are numbered in increasing order from West to East. Tiny will be required to drive around and fire missiles at all of the targets.

Tiny will start in the Northwestern-most cell (in the first row and first column), with his car facing South. Each second, he may either drive to an adjacent cell, or fire a missile in the direction that his car is currently facing. If he chooses to drive, he may do so in one of the four cardinal directions (North, South, East, or West) as long as he stays within the room (he can't drive North from row 1 or West from column 1), and his car will be left facing the direction in which he just drove.

The Head-Monkey will add N ($1 \le N \le 10^5$) targets to the course, one after another, and make Tiny complete the current course after each one is added. Each time, Tiny will have to start in the Northwestern-most cell and hit all of the targets which have been added so far with missiles, in any order. That is, he will have to complete the course N separate times, and on his i-th run, he'll be required to hit targets 1..i. The i-th target's position is described by a character D_i (either "`R`" or "`C`") and an integer P_i ($1 \le P_i \le 10^9$). If D_i = "`R`", then the i-th target is at the far East end of row P_i, meaning that it can be hit by firing a missile Eastward from any cell on row P_i. Otherwise if D_i = "`C`", then the i-th target is instead at the far South end of column P_i. No two targets are at the same location.

To help convince the Head-Monkey of Tiny's driving skills, can you help Tiny determine how quickly the course can be completed each of the N times?

In test cases worth 4/19 of the points, $N \le 10$ and $P_i \le 10$.
In test cases worth another 7/19 of the points, $N \le 40$ and $P_i \le 40$.

Input Format

The first line of input consists of a single integer N.
The next N lines each consist of a character and integer D_i and P_i, separated by a space, for i = 1..N.

Output Format

Output N lines, one integer per line. The i-th line of output (for i = 1..N) should consist of the minimum number of seconds required to complete the course for the i-th time.

Sample Input 1

```
3
R 3
C 2
C 3
```

Sample Output 1

```
4
6
8
```

Sample Explanation 1

The third and final time that Tiny completes the course, one optimal sequence of actions that he can take is as follows:

- Drive East
- Drive South
- Fire a missile (hitting target 2)
- Drive East
- Drive South
- Fire a missile (hitting target 3)
- Drive East
- Fire a missile (hitting target 1)

Sample Input 2

```
2
R 1
R 10
```

Sample Output 2

```
2
13
```

Problem S4: Server Hacking

25 Points / Time Limit: 2.00s / Memory Limit: 32M

The Center for Exploration of Monkey Cryptography (CEMC) is an organization dedicated to wartime cryptography research amongst the primate community. As the monkeys are once again at war with the cows, this institution has become indispensable to their success in combat. After all, taking over Scarberia in the 21st century will require some of the most sophisticated code-breaking techniques in existence to infiltrate the enemies' communications. As it turns out, the cows possess a slight technological edge over the monkeys, due to their experimentation with Cow-Bots in 2002. They were therefore able to reverse engineer the structure of the CEMC's servers!

The cows have learned that the server is a network consisting of N ($1 \le N \le 100{,}000$) computers arranged in a line topology. Specifically, the computers are numbered from 1 to N, and are connected in a line in ascending order. For example, computer 1 is connected only to computer 2, computer 2 is connected only to computers 1 and 3, and computer N is connected only to computer $N - 1$. The network is encrypted using a simple public key cryptosystem. Each computer i has a distinct public key, an integer A_i ($1 \le A_i \le 10^9$). The private key to access computer i is a sequence of integers based on the prime factorization of A_i. Consider $A_i = 1200$ with the prime factorization of $2^4 \times 3^1 \times 5^2$ when the prime factors are listed in ascending order. Its private key is then the sequence [2, 4, 3, 1, 5, 2].

Typical brute force attacks rely on trying every possibility to gain access. In this case, a private key is easier to guess if it comes "earlier" in an exhaustive list of possible guesses. So for two different computers i and j, we will call the public key of i **weaker** than the public key of j if and only if the private key sequence of i is lexicographically smaller than the private key sequence of j. A sequence X is considered to be lexicographically smaller than another sequence Y if $X_i < Y_i$ for the smallest index i such that X_i differs from Y_i. If every value in X is equal to the values at corresponding indices in Y and X is a shorter sequence (i.e. if X is a prefix of Y), then X is also considered lexicographically smaller than Y. For example, [2, 2, 7, 1] is lexicographically smaller than [2, 3] and [2, 3] is lexicographically smaller than [2, 3, 5, 7].

Having discovered the weak cryptographic scheme of the CEMC's servers, the cows now wish to assert their dominance and generate chaos amongst the monkeys by performing a distributed denial-of-service (DDOS) attack on the network to take it down. However, they will need a **weakpoint** from which to initiate the attack. In particular, a weakpoint is a computer whose public key is weaker than that of every other computer that it is directly connected to. Please help the cows write a program to find a weakpoint and crash the CEMC's servers. If there are multiple such weakpoints, then any one of them will suffice.

In test cases worth 5/25 of the points: $N \le 1000$.

Input Format

The first line of input consists of a single integer N.
N lines follow, with the i-th of these lines containing A_i (for $i = 1..N$).

Output Format

Output a single integer between 1 and N, your chosen weakpoint.

Sample Input

```
4
840350
341796875
1584
735166567
```

Sample Output

```
1
```

Explanation

Computer 1's public key has the prime factorization 840350 = $2^1 \times 5^2 \times 7^5$, so its private key is [2, 1, 5, 2, 7, 5].
Computer 2's public key has the prime factorization 341796875 = $5^{11} \times 7^1$, so its private key is [5, 11, 7, 1].
Computer 3's public key has the prime factorization 1584 = $2^4 \times 3^2 \times 11^1$, so its private key is [2, 4, 3, 2, 11, 1].
Computer 4's public key has the prime factorization 735166567 = $26783^1 \times 27449^1$, so its private key is [26783, 1, 27449, 1].
Therefore, both computers 1 and 3 are valid weakpoints that will be accepted.

Problem S5: Supply Chain

33 Points / Time Limit: 5.00s / Memory Limit: 64M

At last, the final battle for Scarberia is upon us! The cows and monkeys have agreed that the fighting will take place on a circle of N ($3 \le N \le 300{,}000$) pastures, connected to one another in a cycle by N two-way bridges. The i-th bridge can support a weight of at most S_i ($1 \le S_i \le 10^6$) pounds, and connects pastures i and $i + 1$ (if $1 \le i < N$), or pastures N and 1 (if $i = N$).

The monkeys' base of operations is located in pasture 1, and they have troops stationed in all of the other pastures. The monkeys, being masterful makers of war, are keenly aware that the most effective soldiers are well-fed soldiers. As such, they have a convoy of M ($1 \le M \le 300{,}000$) trucks which will deliver bananas from their base daily. The i-th truck has a weight of W_i ($1 \le W_i \le 10^6$) pounds, and every day, it will depart from pasture 1, driving around amongst the pastures and delivering B_i ($1 \le B_i \le 10^6$) bananas to each other pasture that it can reach.

Each truck can only cross bridges which can at least support its weight, and it can drive back and forth as much as necessary (even revisiting pastures multiple times), but it will only deliver a load of bananas to each pasture at most once. Therefore, each day, the i-th truck will deliver between 0 and $B_i \times (N - 1)$ bananas in total.

The battle is scheduled to last for D ($1 \le D \le 300{,}000$) days, and in order to manage their banana hoard successfully, the monkeys would like to determine how many bananas their trucks will deliver on each day. However, this aspect of the war promises to be particularly dynamic, as those dastardly cows are also well aware of the military importance of the supply chain and will attempt to sabotage it, while the monkeys will hopefully enact the necessary countermeasures.

One event will occur at the start of each day (before the trucks roll out to make their deliveries, with the type of the event on the i-th day indicated by T_i ($1 \le T_i \le 2$):

- If $T_i = 1$, the cows will undermine the structural integrity of the X_i-th ($1 \le X_i \le N$) bridge, permanently reducing its maximum supported weight by Y_i ($1 \le Y_i < 10^6$) pounds. It's guaranteed that it will still be able to support a weight of at least 1 pound.
- If $T_i = 2$, the monkeys will remodel their X_i-th ($1 \le X_i \le M$) truck such that it henceforth weighs Y_i ($1 \le Y_i \le 10^6$) pounds. Note that this could cause its weight to increase (thanks to cows cleverly disguised as monkey mechanics).

The outcome of the entire war, and the monkeys' opportunity to reclaim Scarberia once and for all, depends on this supply chain! Can you help the monkeys determine how many bananas their trucks will deliver in total on each of the D days of the battle?

In test cases worth 6/33 of the points, $N \le 200$, $M \le 200$, and $D \le 200$.
In test cases worth another 8/33 of the points, $N \le 2000$, $M \le 2000$, and $D \le 2000$.
In test cases worth another 8/33 of the points, $T_i = 1$.
In test cases worth another 8/33 of the points, $T_i = 2$.

Input Format

The first line of input consists of three space-separated integers, N, M, and D.
N lines follow. The i-th of these lines consists of a single integer, S_i (for i = 1..N).
M lines follow. The i-th of these lines consists of two space-separated integers, W_i and B_i (for i = 1..M).
D lines follow. The i-th of these lines consists of three space-separated integers, T_i, X_i, and Y_i (for i = 1..D).

Output Format

Output D lines. The i-th line of output should consist of a single integer representing the total number of bananas delivered on the i-th day (for i = 1..D). Note that each of these values may not fit within a 32-bit integer.

Sample Input

```
4 5 4
5
4
2
8
3 5
1 100
2 1
5 20
6 4
2 2 100
1 4 3
1 4 4
2 2 1
```

Sample Output

```
62
58
33
333
```

Explanation

On the first day, the first truck will deliver 5 bananas each to pastures 2, 3, and 4. The second truck, now weighing 100 pounds, won't deliver any bananas. The third truck will deliver 1 banana to each of those 3 pastures, the fourth will deliver 20 bananas to pastures 2 and 4, and the fifth will deliver 4 bananas to just pasture 4. The total number of bananas delivered on this day will then be 5×3 + 1×3 + 20×2 + 4×1 = 62.